

Chapter 31

Thought Processes of a Mathematician

The Hazards of Playing the Financial Markets

Show that there exists a set A of positive integers with the following property: For any infinite set S of primes there exist two positive integers m in A and n not in A , each of which is a product of k distinct elements of S for some k greater than 1.

William stared at the question, struggling with the grammar. Clearly he was being invited to divide the positive integers into two parts, A and not- A . But what was the criterion for this division? Working on the scratch pad, he tried some possibilities. A the set of all even numbers—of course not. A some subset of the primes . . . no, couldn't be made to work. Then what? He tried negating the problem's statement for a *reductio ad absurdum*, but got lost in the negation logic.

He fretted with the question—it was the first of six on the paper—but could get nowhere with it. He hastened to the next. Two mirrors set at an angle to each other, a ray of light coming in at another angle, how many reflections? His spirits rose. Nothing to it but brute trigonometry! He hacked through it without pausing, but it still took twenty minutes—and a quarter of the three hours was gone! He had wasted twenty-five minutes on that first question!

Gritting his teeth, William attacked the third question. On a first reading it was as opaque as Question One; but he forced himself to read it

through again, calmly and slowly, and saw at once that it was a straightforward exercise in projective geometry, disguised by a lot of superfluous conditions. Four was an ingenious puzzle in number theory that came out in four lines once you saw the trick. Five was a messy business with a cone, a sphere and a cube resting against each other, which could be short-circuited to some degree by a change of coordinate systems. Six was a geometrical trap of great elegance: the unimaginative student forced with calculus formulas would attack it, and eventually solve it, via a chain of integration by parts, but if you just considered it coolly for a moment, and were familiar with a certain ancient puzzle involving tessellations of the plane, it could be solved very easily by applying a series of reflections, without using calculus at all.

Delighted with himself at having seen through this last question, William set his pen down and checked the clock. Fifteen minutes to spare! Glancing round, he saw the other students still working, every one. So! He had finished ahead of them all! Resting back in his seat, he closed the question book, which was actually just a single folded sheet, and let his head fall back, so that he was looking up at the ceiling. To emphasize the effect further, he stretched his arms up vertically, cracking his knuckles, then set his hands behind his head in a posture of relaxation. Should work over his results, perhaps, and see if any could be improved. But he felt sure he had captured the essence of all the questions, and that there was little improvement to be made. He bent forward again, anyway, to review his work . . . and saw Question One laughing back at him, still unanswered!

Frantically William scabbled for his pen, dropped it, picked it up, swept the question paper off his desk while straightening up, picked that up, turned it right way round, and stared sweating at Question One. *Show that there exists a set A . . .* Made up of what? Excluding what? He fumbled with figures on the scratch pad, trying things he knew were hopeless even before the pen touched the paper. Twelve minutes. Eleven. *For any infinite set S of primes there exist . . .* So, if only he could construct this set A , he could face up to anyone challenging him with a set S , defy them, vanquish them! by producing the golden antidotes m and n . Nine min-

utes. The chief invigilator stirred, rising cautiously from his desk to embark on a tiptoe patrol up the center aisle.

In panic and despair William suddenly, in his mind's eye, saw Han Yuezhu. She was grim-faced, angry, accusing, and was thrusting towards him an infinite set S of primes, in the form of a large, heavy stick of dark brown bamboo. The set was right there, cut in rough symbols along the bamboo:

$$S = \{p_1, p_2, p_3, p_4, p_5, p_6, \dots\}$$

Yuezhu was pushing it at him, thrusting it at him, challenging him, her face twisted in hatred, modified now by the beginnings of a triumphant leer. How could he riposte? What did he have to defend himself with? Cornered, despairing, he turned—and saw a long rack, an infinitely long rack, of swords, and a label above each:

$$k = 1 \quad k = 2 \quad k = 3 \quad k = 4 \dots$$

Of course! He had freedom of choice with k ! It could be anything he wanted! No matter what set of primes she challenged him with, there would be a k to meet it! And each k must correspond to a subset of A . . . so A was constructed from subsets . . . each one keyed on what? on what? on the k -th prime, of course!

William wrote a proof out on the scratch pad. It was messy and round-about, but once he saw it in writing all was clear. He condensed it to a dozen elegant lines on the answer sheet . . . then saw that it could even be cut down to half that, if only . . .

DZZZZZZZZZZZZ!

“Pens down, please, ladies and gentlemen,” said the invigilator, who was English. “At *once*, please, young lady,” (to a recalcitrant examinee). “Thank you.”

He gestured to his minions at the sides of the room. They advanced along the outward aisles, collecting papers. William took a last regretful glance at his solutions before the boy took them. Those superfluous lines in the proof! Another thirty seconds and he could have got rid of them.

That was the kind of thing that might make all the difference. But life had taught William sufficient fatalism. Putting the examination out of his mind, He left the hall and walked over the hills to Aberdeen, taking Halton Road past the Peak.

When Mr Ng got home from work he insisted on seeing the question paper, which William had brought out with him. Mr Ng scrutinized the paper in silence for several minutes, sitting at the circular fold-up dinner table, sipping at his glass of beer.

“Ha!” he said at last.

“Ha, what?”

“Ha, I can’t find even one sentence that makes sense.”

“But you can hardly read English, Uncle.”

“I can read well enough. But *this* . . . Is it really useful?”

William laughed. “I don’t know. Maybe not.”

“Such things in the world. You live your whole life not knowing them.” Mr Ng shrugged, and looked straight at William. “Did you answer all the questions?”

“Yes. Every one.”

Mr Ng nodded, and dropped his eyes. “But you mustn’t get your hopes too high, you know. If you can go to the University, that will be wonderful. But don’t hope too much. Hope is a drug, you know. Like liquor, like opium. Feels very nice while you enjoy it. Afterwards—big hangover.” He held William’s eyes again.

William laughed. “Don’t worry, Uncle. I did my best, that’s all. Nobody can do more.”

Mr Ng nodded and sighed. “You have so much ability. I’m sorry . . .” he glanced across the table at William again . . . “I’m sorry I haven’t the resources to give you the education you deserve. As things are, everything is up to you.”

“It’s all right, Uncle. You’ve done everything you can for me. I have no complaints.”

Old Ng considered this for a while, sipping his San Miguel beer. Then, without looking up: “You’re a good boy, Wailam.”

William wished he could do something for the Ngs. They had been so kind to him. It seemed wrong that such good people should have so

little. He thought of Fourth Outside Uncle. There was a man with everything, yet he could turn away a penniless relative from his door. Human nature was really unfathomable.

This was in early December, William's second and last year at the school in Aberdeen. He wanted to go to college, but the Ngs could not afford it. William especially wanted to go to Hong Kong University, the colony's premier institution. His only chance was to get a scholarship; and there was precisely one full scholarship offered in mathematics, awarded by competitive examination, the examination William had just sat for.

Since the events of the summer he had withdrawn from his classmates altogether and buried himself in study. The school had no library, but William had joined the main Hong Kong public library over in Central district, on the other side of the island. You can walk right over the spine of the Hong Kong island, from Aberdeen to Central, along quiet roads and tracks that avoid the Peak, and offer spectacular views of the colony, the mainland, and the South China Sea. William often took this walk on a Saturday morning, over the hills and down into the clamor of Central.

The other pole of his solitary world now was the University bookstore on Pokfulam Road. The general library in Central had few books on higher math, and he had long since absorbed them all. The University bookstore had a much richer collection. He could not afford to buy them, of course—the cheapest were forty or fifty dollars—but he could browse as much as he liked, and in this way he managed to get all he needed, working out the exercises in his head, or memorizing them to take home and attack with paper and pencil.

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A few days after the examination William went to the University bookstore to browse. The problem of the tessellations had stuck in his mind, and he had worked out a generalization, but needed to know more about Group Theory. However, none of the books was helpful. William's mind wandered, and he drifted aimlessly for a while among the shelves.

There was a section, separate from the main mathematics books, of almanacs, ephemerides, statistical and nautical tables. These had a certain fascination for William. They reminded him of Abramovitz and Stegun on Father's bookshelf, in another world long ago. Pulling down these books at random to savor the numbers all in their ranks and files, William found himself looking at some bond tables. He had no idea what a bond was, much less any inkling of rates or yields; but he at once saw the smooth regularity in the numbers, and mentally sketched a curve to fit them. The book was a compendium of business statistics, including long lists of prices of various kinds of securities and commodities on the exchanges of the world, up to the year 1969. Some of the numbers flowed regularly on simple curves, like the bond yields; others seemed to fluctuate without rhyme or reason.

The endless columns of numbers, and their odd, tantalizing patterns of behavior, caught William's interest. He hiked round to Central and found the same book in the library's reference section. William took the book back to a desk and scanned through the pages. The commodity and currency prices seemed to be the most irregular. Working mentally—he had no writing materials—he tried doing first and second differences, but the randomness just increased. Irritated, he tried comparing different commodities over the same time periods. This was more fruitful: there were patterns of reflection and inversion. Thoroughly captured now, William begged a pencil and some paper from the librarian, and settled down with the columns of numbers.

At dinner that evening he asked Mr Ng about commodities. He didn't know the Chinese word, and Mr Ng didn't know the English, so they had to look it up.

"I know," said Mr Ng. "There's an exchange. Like the stock exchange. Business people buy and sell those things. The exchange regulates it."

"Can anybody do it? Buy and sell, I mean."

"I don't think so. You need a pot of money, anyway."

"A broker," said Mrs Ng. "You need a broker."

William and Mr Ng stared at her. "Since when do you know about these things?" asked Mr Ng.

“Old Mother Lo in 621 plays the stock market. She told me all about it.”

Mr Ng stared at his wife. “Old Mother Lo? She hasn’t got two pennies to rub together.”

Mrs Ng laughed merrily, delighted with her secret. “She’s a miser. Her husband used to live in Canada, you know. He had money saved. She really shouldn’t be living in public housing at all.”

“This broker,” asked William. “Who is it? What does he do?”

“He handles the buying and selling. Ordinary people aren’t allowed to. You have to have a license. Well, he has a license. He lends you money, too. To buy the stocks. It’s called *trading on margin*.”

“Since you know all this stuff, it’s surprising we’re not rich,” sniffed Mr Ng.

“We never have any money,” laughed Mrs Ng. “What would we buy with?”

“But you said the broker lends you money,” put in William.

“Mmm, yes. But still you must put up a deposit. I can find out if you like.”

They sent her off to consult with Old Mother Lo. When she was gone, Mr Ng asked: “Why do you want to know about commodities and brokers?”

“I was reading a book in the library, about commodities. The prices of different ones are related in some way. It’s complicated, but I can figure it out. If you watch what one is doing today, sometimes you can tell what another one will do tomorrow. Gold and platinum, for example. If gold goes up more than three days in a row . . .”

He tried to explain to Mr Ng, but could see he wasn’t doing very well. It was hard to express without using mathematics. Just as Mr Ng’s perplexity seemed to be crossing over into actual physical pain, Mrs Ng came back with Old Mother Lo.

“It’s a thousand dollars minimum,” said Old Mother Lo. “They won’t deal with you for less than that. But if you put up a thousand you can trade for ten thousand. They take a cut, of course.” She gave them a long lecture on stock trading, but seemed to know nothing about commodities.

They gave Old Mother Lo some sticky rice cakes and sent her home. “I don’t know why we’re talking about this,” said Mrs Ng when the old woman had gone. “We’re never going to have a thousand dollars.”

“Might be able to raise it,” said Mr Ng, looking at William as he spoke. “I haven’t asked my family for anything for a long time. And your people . . . well, there was that loan last summer. But we repaid that at New Year. Won’t hurt to ask.”

“You’re crazy!” Mrs Ng laughed at him. “Playing the stock market with borrowed money! Crazy! We’ll be in debt all our lives!”

“It’s not stocks, it’s commodities. And young Wailam here seems to know what to do.” Mr Ng had not taken his eyes off William. “Don’t you?”

“I think so. But can we really get a thousand dollars?”

“If it’s a sure thing I can get it. Is it a sure thing?”

“Maybe. I’m really not certain. I need to read some more.”

“A sure thing but not certain!” Mrs Ng reached across the table and slapped her husband playfully on the cheek. “Wake up, old turtle! We’re not business people! We’re like donkeys and oxen, born to work.”

“Let him do his research. If he says it’s a sure thing, I’ll believe him.”

And so William went into the commodities markets. Mr Ng made the rounds of the relatives, and somehow raised a thousand Hong Kong dollars—a hundred and seventy U.S. at this point in time. William was too young to have an account at the brokers, but Old Mother Lo introduced Mr Ng, and the broker took him on. William watched the price of gold, matching the patterns. On the appropriate day, he told Mr Ng to buy platinum futures. They put in the entire one thousand, on margin, and tracked the investment in Mr Ng’s daily newspaper.

“Can’t see much movement,” complained Mr Ng. “It’s inching up, but hardly enough to cover the commission.”

“Wait,” said William. “Wait.”

They waited. Gold hesitated, as William had expected it to; then sagged. Silver trailed gold. Platinum turned up, climbed, then soared.

“Get out now,” said William. “First thing in the morning.”

They had made a sixty per cent profit in a month. Mr Ng got the brokers to write out a statement for him, and brought it home, and set it on the dining table where they could all wonder at it.

“Five thousand eight hundred dollars,” said Mr Ng. “I have never had so much money in my life. Never *seen* so much.”

“It’s a miracle,” said Mrs Ng. “But if you won all this, somebody must have lost it.”

“I can do it any time,” said William.

His second venture, however, was less successful. He had developed a complicated rule he called the W Theory. According to this, when the price of gold had executed a symmetrical W shape on one of his time series, silver would fall and then rise. This worked, but not as well as he had hoped, and their profit this time was just four per cent. Still, Mr Ng’s confidence in him was now total.

“If we can play a few more like this, you’ll be able to go to the University after all,” said Mr Ng.

William tried a new, vastly more complicated ploy: returning to platinum, but with cross-bets on silver, gold, and three different currencies. This time he had to monitor the trades himself, as Mr Ng had been unable to grasp the strategy. Day by day William checked the listings in the *South China Morning Post*. The Yen and Deutschmark rose against the dollar, then parted company in just the even, symmetrical way he wanted, like the opening petals of a flower on his graph. When their curves had diverged to a pretty cornet shape, he sold silver and bought gold. When his figures summed to a certain figure, he sold Deutschmarks and gold, bought dollars and platinum. Platinum obediently rose, rose, rose—not by any means beyond his wildest dreams, but strongly, sufficiently. It would rise through to the end of the week, he knew.

Or thought he knew. On Wednesday platinum stalled; on Thursday it dropped like a stone. Mr Ng enjoyed his first margin call, or would have if he had been at home to take it. Not being at home, his positions—futures and currencies all—were liquidated.

“Big new platinum find in central Africa.” (This was the broker, when Mr Ng and William went there on Friday morning.) “The French cut a deal with the government. Mining will start soon. Demand pretty fixed, you know. Get a new supply, prices gonna drop. These things happen.”

It could have been worse. The silver trades, plus the appreciation of the Yen, had almost covered their net deficit. By the time they walked out

of the broker's office they knew that they were only four hundred dollars poorer than when they had started six weeks before, except that the thousand dollars Mr Ng had borrowed from his relatives had disappeared into the futures market along with all their winnings.

"I'm sorry," said William, as they walked down Nathan Road to the Star Ferry. The broker was on Kowloon side. "Really sorry."

"Guess it's true what people say," said Mr Ng: "The markets are really a lottery."

"I'll get it back," said William. "I've got a new theory . . ."

"Using what?" Mr Ng wasn't mad, but he wasn't smiling, either. "We've lost our funds. I owe a thousand to my cousins and four hundred to the broker. Not to mention lost wages from taking the morning off work. No, let's learn our lesson. No more markets. We are small people, working people. We shouldn't get involved in these rich man's games."

"But then how can we pay back what we owe?"

"Oh, we'll go short for a while." Mr Ng shrugged.

"I'll get a job after school."

"No, you won't." Mr Ng stopped, and faced William. "This is not your fault, don't feel bad. I'm the head of my family. I took the decision to invest. I had too much faith in you. No, not in you, William, in your mathematics. Seeing that examination paper you brought home, I thought, wa!—a person who can understand this stuff can surely crack the stock market. But the market is a worldly thing, and mathematics is a mental thing. I didn't think of that. You're just a kid, you don't know anything about the world. No-one could expect you to think of it. But me, I really should have. A new platinum mine in Africa! Who could predict it? How could your mathematics tell you that? So I didn't think things through as I should have done. Now I am much wiser. It cost me fourteen hundred dollars, that's all. But you know . . ." He grinned up at William . . . "wisdom is cheap at any price." He laughed—freely, genuinely, at himself. "So don't worry about the fourteen hundred. We'll get it together somehow. But no more market games!"

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Back on Hong Kong side they went their ways, Mr Ng to his job in North Point, William back to Aberdeen for school. The bus went along Pokfulam Road past the University. Little hope now of studying there. That damn Question One! It was his fault, like the fiasco with commodities. He had been overconfident. If only he had gone straight back to it, in that extra couple of minutes he could have perfected his proof. After all, who was he to be so confident? A country boy from a town no-one had heard of. A vagabond, a thief, bum-boy to a foreign devil, plaything of an *acha* nymphomaniac.

William reached school overcome with shame and self-doubt. He had intended to slip into the last class of the morning, which was English. However, one of his classmates met him at the lab door.

“Wailam, where have you been? The Principal wanted to see you.”

So now he was to be scolded for skipping classes. Well, it was no more than he deserved. Resigned to his fate, William made his way to the Principal’s office. One of the secretaries sent him straight through. The Principal was a small, clerkish fellow called Kong. Somewhat to William’s surprise, he stood up as William entered, grinning unnaturally from behind his thick glasses.

“Well, well! Congratulations!”

Taking this as sarcasm, William momentarily wondered if Principal Kong had got wind of his speculations on the commodities exchange.

“Our star pupil! Such an honor for our school!”

Principal Kong had come out from his desk to shake William’s hand. William was baffled. This did not seem like sarcasm. Kong shook William’s hand with both of his, like an American.

“What . . . I’m sorry. I know I was late in today, but . . .”

Principal Kong raised his eyebrows and stepped back. “Did Mr Cheui not tell you? I thought he had told you. Then it is my pleasure to tell you!” He was beaming, clearly delighted about something.

“Tell me?”

“You have won the math scholarship to Hong Kong University! You placed first in the whole territory!” (Hong Kong people, speaking in Cantonese, never said “colony”.)

“Really? Oh!”

“Not only that, but you got a perfect score. One hundred per cent! The first time in twenty-seven years! It’s a sensation! Perfect score! Such an honor for our school!”

Mr Ng smiled when William told him, and nodded his satisfaction. “Sak Ngong lost his horse,” he said. “Sak Ngong lost his horse.” This idiom referred to an old story about a man named Sak Ngong—“Sai Weng” in Mandarin.

Sai Weng Lost His Horse

Once there was a man called Sai Weng who lived near the Wall. One day he lost a horse. His neighbors expressed their sympathy, but Sai Weng only said: “Human beings cannot understand the play of fortune and misfortune. It is too complex. Perhaps some good will come from this.”

After a few days his horse came back, bringing with it a herd of wild horses from the grasslands of the west. Sai Weng captured and tamed the wild horses. His neighbors said: “What good fortune! With these horses, you can be considered a rich man. It must be a gift from Heaven!” Sai Weng only said: “Who can tell? Perhaps some evil will come from this.”

Sure enough, one of the horses, whose blood was still wild, threw Sai Weng’s son. The son broke a leg and became lame. When Sai Weng’s neighbors came to offer condolences, he only said: “Nobody can understand the ways of Heaven. Perhaps some good will come of this.”

Soon after that, barbarians broke through the Wall. All the young men of the district were conscripted to fight them, and all the young men were killed. Sai Weng’s son, however, was not conscripted because he was lame. So of all the people in his district, only Sai Weng had a son to care for him in old age.

At the University William was interviewed by four members of faculty in a pleasant little sitting-room overlooking the campus. They intro-

duced themselves to him one by one, coming up out of their armchairs to shake hands, all of them addressing him in English. Three of them were Europeans, one Chinese. The Chinese, and one of the Europeans, seemed to be full Professors.

“You gave a very impressive performance, young man,” said Professor Meld, when everyone was seated again.

“Avoided all our little traps,” chuckled Lecturer Moore.

“You were the only one who made a dent on Question One,” said Lecturer Michaels. “Nobody else got anywhere with it. The *pons asinorum*.”

“And your change of coordinates,” said Professor Cheung. “I had not thought of it myself, until I saw your solution.”

“Thank you,” said William. “Thank you.”